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Probability Theory and Applications (MA208)
Problem Sheet - 8

The Poisson and Other Discrete Random Variables

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1. If X has a Poisson distribution with parameter β, and if $P(X = 0) = 0.2$, evaluate $P(X > 2)$.

2. Let X have a Poisson distribution with parameter λ. Find that value of k for which $P(X = k)$ is largest. [*Hint:* Compare $P(X = k)$ with $P(X = k - 1)$.]

3. (This problem is taken from *Probability and Statistical Inference for Engineers* by Derman and Klein, Oxford University Press, London, 1959.) The number of oil tankers, say N, arriving at a certain refinery each day has a Poisson distribution with parameter $\lambda = 2$. Present port facilities can service three tankers a day. If more than three tankers arrive in a day, the tankers in excess of three must be sent to another port.
   (a) On a given day, what is the probability of having to send tankers away?
   (b) How much must present facilities be increased to permit handling all tankers on approximately 90 percent of the days?
   (c) What is the expected number of tankers arriving per day?
   (d) What is the most probable number of tankers arriving daily?
   (e) What is the expected number of tankers serviced daily?
   (f) What is the expected number of tankers turned away daily?

4. Suppose that the probability that an item produced by a particular machine is defective equals 0.2. If 10 items produced from this machine are selected at random, what is the probability that not more than one defective is found? Use the binomial and Poisson distributions and compare the answers.

5. An insurance company has discovered that only about 0.1 percent of the population is involved in a certain type of accident each year. If its $10,000$ policy holders were randomly selected from the population, what is the probability that not more than 5 of its clients are involved in such an accident next year?

6. Suppose that X has a Poisson distribution. If
$$P(X = 2) = \frac{2}{3}P(X = 1),$$
evaluate $P(X = 0)$ and $P(X = 3)$.

7. A film supplier produces 10 rolls of a specially sensitized film each year. If the film is not sold within the year, it must be discarded. Past experience indicates that D, the (small) demand for the film, is a Poisson-distributed random variable with parameter 8. If a profit of \$7 is made on every roll which is sold, while a loss of \$3 is incurred on every roll which must be discarded, compute the expected profit which the supplier may realize on the 10 rolls which he produces.

8. Particles are emitted from a radioactive source. Suppose that the number of such particles emitted during a one-hour period has a Poisson distribution with parameter λ. A counting device is used to record the number of such particles emitted. If more than 30 particles arrive during any one-hour period, the recording device is incapable of keeping track of the excess and simply records 30. If Y is the random variable defined as the number of particles *recorded* by the counting device, obtain the probability distribution of Y.

9. Suppose that particles are emitted from a radioactive source and that the number of particles emitted during a one-hour period has a Poisson distribution with parameter λ. Assume that the counting device recording these emissions occasionally fails to record an emitted particle. Specifically, suppose that any emitted particle has a probability p of being recorded.

   (a) If Y is defined as the number of particles recorded, what is an expression for the probability distribution of Y?

   (b) Evaluate $P(Y = 0)$ if $\lambda = 4$ and $p = 0.9$.

10. Suppose that a container contains $10,000$ particles. The probability that such a particle escapes from the container equals 0.0004. What is the probability that more than 5 such escapes occur? (You may assume that the various escapes are independent of one another.)

11. Suppose that a book of 585 pages contains 43 typographical errors. If these errors are randomly distributed throughout the book, what is the probability that 10 pages, selected at random, will be free of errors? [*Hint:* Suppose that $X =$ number of errors per page has a Poisson distribution.]

12. A radioactive source is observed during 7 time intervals each of ten seconds in duration. The number of particles emitted during each period is counted. Suppose that the number of particles emitted, say X, during each observed period has a Poisson distribution with parameter 5.0. (That is, particles are emitted at the rate of 0.5 particles per second.)

   (a) What is the probability that in each of the 7 time intervals, 4 or more particles are emitted?

   (b) What is the probability that in at least 1 of the 7 time intervals, 4 or more particles are emitted?

13. It has been found that the number of transistor failures on an electronic computer in any one-hour period may be considered as a random variable having a Poisson distribution with parameter 0.1. (That is, on the average there is one transistor failure every 10 hours.) A certain computation requiring 20 hours of computing time is initiated.

   (a) Find the probability that the above computation can be successfully completed without a breakdown. (Assume that the machine becomes inoperative only if 3 or more transistors fail.)

   (b) Same as (a), except that the machine becomes inoperative if 2 or more transistors fail.

14. In forming binary numbers with n digits, the probability that an incorrect digit will appear is, say 0.002. If the errors are independent, what is the probability of finding zero, one, or more than one incorrect digits in a 25-digit binary number? If the computer forms 10^6 such 25-digit numbers per second, what is the probability that an incorrect number is formed during any one-second period?

15. Two independently operating launching procedures are used every week for launching rockets. Assume that each procedure is continued until it produces a successful launching. Suppose that using procedure I, $P(S)$, the probability of a successful launching, equals p_1, while for procedure II, $P(S) = p_2$. Assume furthermore, that one attempt is made every week with each of the two methods. Let X_1 and X_2 represent the number of weeks required to achieve a successful launching by means of I and II, respectively. (Hence X_1 and X_2 are independent random variables, each having a geometric

distribution.) Let W be the minimum (X_1, X_2) and Z be the maximum (X_1, X_2). Thus W represents the number of weeks required to obtain a successful launching while Z represents the number of weeks needed to achieve successful launchings with both procedures. (Thus if procedure I results in $\overline{S}\,\overline{S}\,\overline{S}S$, while procedure II results in $\overline{S}\,\overline{S}S$, we have $W = 3$, $Z = 4$.)

(a) Obtain an expression for the probability distribution of W. [*Hint:* Express, in terms of X_1 and X_2, the event $\{W = k\}$.]

(b) Obtain an expression for the probability distribution of Z.

(c) Rewrite the above expressions if $p_1 = p_2$.

16. Four components are assembled into a single apparatus. The components originate from independent sources and $p_i = P(i\text{th component is defective})$, $i = 1, 2, 3, 4$.

    (a) Obtain an expression for the probability that the entire apparatus is functioning.

    (b) Obtain an expression for the probability that at least 3 components are functioning.

    (c) If $p_1 = p_2 = 0.1$ and $p_3 = p_4 = 0.2$, evaluate the probability that exactly 2 components are functioning.

17. A machinist keeps a large number of washers in a drawer. About 50 percent of these washers are $\frac{1}{4}$ inch in diameter, about 30 percent are $\frac{1}{8}$ inch in diameter, and the remaining 20 percent are $\frac{3}{8}$ inch in diameter. Suppose that 10 washers are chosen at random.

    (a) What is the probability that there are exactly five $\frac{1}{4}$-inch washers, four $\frac{1}{8}$-inch washers, and one $\frac{3}{8}$-inch washer?

    (b) What is the probability that only two kinds of washers are among the chosen ones?

    (c) What is the probability that all three kinds of washers are among the chosen ones?

    (d) What is the probability that there are three of one kind, three of another kind, and four of the third kind in a sample of 10?

18. Prove Theorem 8.4.

19. Prove Theorem 8.6.

20. The number of particles emitted from a radioactive source during a specified period is a random variable with a Poisson distribution. If the probability of no emissions equals $\frac{1}{3}$, what is the probability that 2 or more emissions occur?

21. Suppose that X_i, the number of particles emitted in t hours from a radioactive source, has a Poisson distribution with parameter $20t$. What is the probability that exactly 5 particles are emitted during a 15-minute period?

22. The probability of a successful rocket launching equals 0.8. Suppose that launching attempts are made until 3 successful launchings have occurred. What is the probability that exactly 6 attempts will be necessary? What is the probability that fewer than 6 attempts will be required?

23. In the situation described in Problem 8.22, suppose that launching attempts are made until three *consecutive* successful launchings occur. Answer the questions raised in the previous problem in this case.

24. Consider again the situation described in Problem 8.22. Suppose that each launching attempt costs \$5000. In addition, a launching failure results in an additional cost of \$500. Evaluate the expected cost for the situation described.

25. With X and Y defined as in Section 8.6, prove or disprove the following:

$$P(Y < n) = P(X > r).$$

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